

# Effects of Cosmic String Velocities and the Origin of Globular Clusters

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With the hypothesis that cosmic string loops act as seeds for globular clusters in mind, we study the role that velocities of these strings will play in determining the mass distribution of globular clusters. Loops with high enough velocities will not form compact and roughly spherical objects and can hence not be the seeds for globular clusters. We compute the expected number density and mass function of globular clusters as a function of both the string tension and the peak loop velocity, and compare the results with the observational data on the mass distribution of globular clusters in our Milky Way. We determine the critical peak string loop velocity above which the agreement between the string loop model for the origin of globular clusters (neglecting loop velocities) and observational data is lost.

## I. INTRODUCTION

We have recently [1] made the hypothesis that string loops arising from a scaling network of cosmic strings seed the formation of the globular clusters which are observed to be distributed in the halos of galaxies, in particular our own Milky Way galaxy. Our model easily explains the observational facts that globular clusters are the oldest and most compact star clusters in our galaxy, and that they are distributed throughout the halo as opposed to only in the disk.

Cosmic string loops arise via the interaction of infinite string segments which in turn are generated during a symmetry breaking phase transition in the early universe. Cosmic strings are predicted in a large class of models of particle physics beyond the *Standard Model*. According to the cosmic string scaling solution [2], the distribution of infinite string segments is independent of the cosmic string tension  $\mu$  (which is, in the natural units we use, equal to the mass per unit length). As a consequence, cosmic string loops are also formed with a number density which is independent of  $\mu$ . The distribution of string loops is determined by the string scaling solution, and depends on  $\mu$  only through the dependence on  $\mu$  of a critical loop radius  $R_c$  below which the number distribution of strings becomes constant [3].

In our previous paper [1] we have shown that if we fix the one free parameter in our theoretical model, namely the string tension  $\mu$ , then the peak number density and the mass distribution are fixed. Demanding that the mass distribution peaks at a value corresponding to the peak in the observed mass function of globular clusters in the Milky Way gave us a value of  $G\mu \sim 10^{-9.5}$  (where we - as is standard in the cosmic string literature - multiplied  $\mu$  by Newton's gravitational constant  $G$  in order to obtain a dimensionless number), a value which is below the current upper bound on  $G\mu$  of  $G\mu < 1.5 \times 10^{-7}$  [4] (see also [5] for earlier work on limits on the string tension). At this point, our string model had no more free

parameters. Interestingly, we found good agreement between the predicted and observed globular cluster mass functions.

However, in our previous study [1] we neglected the presence of string loop velocities (center of mass string loop velocities). Recent numerical simulations (eg. [6]) tell us that loops are typically born with translational velocities that are sizable fractions of the speed of light. The reason is that, since long string segments usually have relativistic speeds, then as string loops split off, the loops also gain significant velocities. When velocities are taken into considerations, accretion will not be spherically symmetric. Additionally, accretion onto a moving loop may be less efficient compared to accretion onto a stationary loop. It should be noted, however, that loop velocities also undergo red-shifting, and thus slow down as the loops age.

In the following, we first review the hypothesis that globular clusters may be seeded by the cosmic string loops that arise from a string scaling solution in particle physics models with a vacuum manifold which has the topology of a circle [1]. In section III we present our a first analysis of velocity effects on our model for the mass function of globular clusters. We compute a suppression factor which takes into account that loops with too large initial velocities will move a distance greater than the size of the spherical object which a stationary loop would accrete, and we incorporate this factor into the predicted overall number density to determine a new mass function. This is then compared with the observed mass distribution of globular clusters in our Milky Way galaxy. Moving loops in fact do accrete matter, but do not give rise to a spherical distribution. In Section IV we compare the total mass from spherical and non-spherical accretion, and determine an alternative criterium for the maximal velocity of a loop that will seed a globular cluster. The resulting mass function turns out to be similar to the one derived using the first criterium. We then turn to a brief discussion of the cosmic string *rocket effect*, another effect neglected in our previous work. We show that this effect has a negligible impact on our globular cluster study. Finally, we present our conclusions in section VI.

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## II. GLOBULAR CLUSTERS FROM COSMIC STRING LOOPS

In this report, we will consider a one-scale model for the distribution of strings which [7, 8] implies that loops of initial radius  $R_i$  form at an initial time  $t_i$  given by

$$t_i = \alpha^{-1} \beta R_i. \quad (1)$$

Here,  $\alpha$  is a constant obtained from numerical solutions [6]. We will use the value  $\alpha = 0.3$ . The average length of a string loop is given by  $l = \beta R$ , where  $\beta$  is taken to be 10 here; if the string loops were exactly circular  $\beta$  would be  $2\pi$ . Making use of the fast decay approximation that assumes loops decay virtually instantaneously, we have  $R(t) = R_i(t_i)$  until the decay time. String loops oscillate and decay by the emission of gravitational radiation [9] whose strength is parametrized by a dimensionless constant  $\gamma$ . Loops with radius smaller than

$$R_c(t) = \gamma G \mu t \quad (2)$$

live less than one Hubble expansion time before decaying. Hence, the number density of loops is constant for  $R < R_c(t)$ .

Linear cosmological perturbation theory tells us that accretion of matter around a cosmic string loop starts at  $t_{eq}$ , the time of equal matter and radiation. At this time, the mass function of the string loop with respect to radius is dominated by loops with a critical radius  $R_{c1}$ , given by

$$R_{c1} = \gamma G \mu t_{eq}. \quad (3)$$

We will use  $\gamma = 10^2$ . Though these cosmic string loops will have decayed by the present time, the objects they seed will continue to grow. The number density at a time  $t > t_{eq}$  for loops formed in the radiation dominated era of such objects inside a galaxy is given by (see e.g. [10] for reviews on the applications of cosmic strings in cosmology)

$$\begin{aligned} n(R, t) &= N \alpha^{5/2} \beta^{-5/2} t_{eq}^{1/2} t^{-2} R^{-5/2} \quad \text{for } R > R_{c1} \\ n(R, t) &= N \alpha^{5/2} \beta^{-5/2} \gamma^{-5/2} (G \mu)^{-5/2} t_{eq}^{-2} t^{-2} \\ &= \text{const.} \quad \text{for } R < R_{c1} \end{aligned} \quad (4)$$

The constant  $N$  depends on the square of the average number  $\tilde{N}$  of long string segments per Hubble volume since two long string segments are required to form a loop. We will take  $N = 10^2$ . Incorporating the Zel'dovich approximation [11], the local number density of string loops inside a galaxy will be enhanced by a factor of  $F$ . This factor is estimated to be  $F = 64$  due to accretion and virialization in each direction. In our calculations, we will use  $F = 10^2$ .

Assuming accretion continues to the present time, the mass which has accreted about these seed loops at the

present time  $t_0$  is given by

$$M(R_{c1}, t_0) = \beta \gamma (G \mu)^2 z_{eq}^{-1/2} \left( \frac{t_0}{G} \right). \quad (5)$$

To obtain a feeling for the meaning of this expression, let us insert the values of  $t_0$  and  $G$ . We then have

$$\frac{t_0}{G} \sim 10^{23} M_\odot, \quad (6)$$

where  $M_\odot$  stands for the solar mass.

In our analysis, we take the peak mass  $M_c$  (the mass where the observed globular cluster mass function for our Milky Way galaxy peaks) to fix our only free parameter, the string tension. The mass function scales as  $M^{-5/2}$  for  $M > M_c$  (which follows directly from the string loop distribution (4)). We predict a linear decay for  $M < M_c$ . This comes about since the loop radius distribution is constant and loops with radius smaller than  $R_{c1}$  live only a fraction of a Hubble time step which scales linearly with  $R$ .

In Fig. 1, a comparison of the predicted mass function (the solid lines) with the observed distribution of globular clusters in the Milky Way (histogram values) compiled from [12] is made. To obtain the theoretical curve, we take the comoving number density  $n(R, t_{eq}) z_{eq}^3$  of loops (where  $n(R, t)$  is given in (4)), multiply the result by the concentration factor  $F$ , allow each loop to grow in mass by a factor of  $z_{eq}$  (independent spherical accretion), and convert this into a mass distribution  $n(M, t_0)$ , while taking into account the Jacobian of the transformation from  $R$  to  $M$ . The result is then multiplied by the bin size  $\delta M = f M$ , where  $f$  is a number, and by the volume  $V$  of the Milky Way galaxy. We obtain the following peak number density bin using (5)

$$\delta N = N F f \alpha^{5/2} \beta^{-5/2} \gamma^{-3/2} (G \mu)^{-3/2} z_{eq}^{3/2} t_0^{-3} V, \quad (7)$$

where  $z(t)$  is the cosmological redshift at time  $t$ .

All of the calculations summarized in this section assumed that cosmic strings loops are created and remain at rest. In the following two sections, we will study the effect of velocities (translational center of mass motion) through two different analyses.

In our first analysis (discussed in the following section), we will compute the mass function of objects which accrete onto loops with velocities low enough such that the loop center moves a smaller physical distance than the physical radius of the shell which would be collapsing onto the loop if it had been stationary. If the loop moves further than this, we assume that no globular clusters forms.

In our second analysis (to be discussed in the next to following section) we study the accretion of matter onto moving loops and keep only objects which are sufficiently spherical. We find that both conditions give similar resulting mass functions for globular clusters, the one from the second condition being slightly higher.

In the following analyses, we are neglecting the rocket

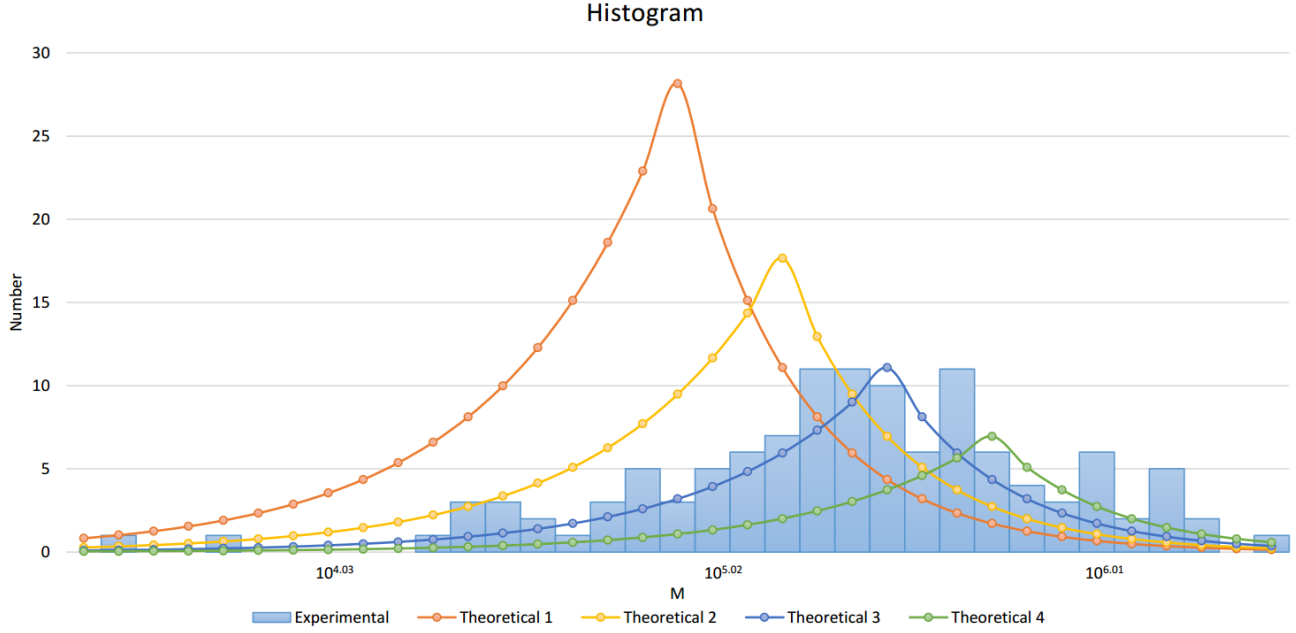


FIG. 1. Dependence of the mass function of our model on the string tension  $G\mu$ . The horizontal axis is mass on a logarithmic scale, the vertical axis gives the number density on a linear scale. The histogram shows the data taken from [12]. The curves shown are for  $G\mu = 2.92 \times 10^{-10}$ ,  $G\mu = 3.98 \times 10^{-10}$ ,  $G\mu = 5.43 \times 10^{-10}$  and  $G\mu = 7.41 \times 10^{-10}$  (in increasing order of mass at the peak position). The blue solid curve minimizes  $\chi^2$ . The cosmic string parameters chosen are described in the text. The effects of string velocities are neglected.

effect which is the effect of anisotropy in the loop gravitational radiation that causes loops to accelerate as they decay. We show in section V that this is indeed a good approximation.

### III. EFFECT OF COSMIC STRING VELOCITY: FIRST ANALYSIS

Accretion onto a cosmic string loop can be studied using the Zel'dovich approximation. As shown in e.g. in [13], the physical distance  $h(q, t_0)$  from the center of a string loop to the mass shell which is “turning around” (i.e. becoming gravitationally bound) at the present time  $t_0$  is given by

$$h(R, t_0) = \left(\frac{9}{5}\right)^{1/3} \beta^{1/3} (G\mu)^{1/3} z_{eq}^{1/3} t_0^{2/3} R^{1/3}. \quad (8)$$

On the other hand, for an initial physical velocity  $v_i$ , the distance a loop of radius  $R$  has moved by the present time is

$$\begin{aligned} \Delta r(R) &= a(t_0) \int_{t_{eq}}^{t_0} \left(\frac{a(t_i)}{a(t)}\right)^2 \frac{v_i}{a(t_i)} dt \\ &= 3\alpha^{-1/2} \beta^{1/2} z_{eq}^{1/4} t_0^{1/2} R^{1/2} v_i. \end{aligned} \quad (9)$$

In the present analysis we will only count the number

of string loops for which

$$\Delta r(R) < h(R, t_0), \quad (10)$$

and we assume that the accretion onto faster moving loops is not effective at producing compact globular clusters. Making use of (8) and (9) we obtain an upper bound on the initial velocity:

$$v_i < \left(\frac{1}{15}\right)^{1/3} \alpha^{1/2} \beta^{-1/6} (G\mu)^{1/3} z_{eq}^{1/12} t_0^{1/6} R^{-1/6}. \quad (11)$$

Taking the distribution of initial velocities in each of three spatial directions to be a step function of width  $v_{\max}$  leads to the following probability that a string loop will satisfy the condition (11):

$$\mathcal{P}(v) = \frac{1}{3} v_i^3 v_{\max}^{-3}, \quad (12)$$

where  $v_{\max}$  is the maximum velocity of cosmic strings determined through cosmic string evolution simulations. Taking the integral of the velocity distribution from zero to the upper bound on  $v_i$ , the rate of globular cluster formation becomes suppressed by a multiplicative factor  $\mathcal{S}(R)$ :

$$\mathcal{S}(R) = \frac{1}{45} \alpha^{3/2} \beta^{-1/2} (G\mu) z_{eq}^{1/4} t_0^{1/2} v_{\max}^{-3} R^{-1/2}. \quad (13)$$

The distribution has a cutoff when  $\mathcal{S}(R) = 1$  leading to

the critical radius  $R_{c_2}$ :

$$R_{c_2} = \left(\frac{1}{45}\right)^2 \alpha^3 \beta^{-1} (G\mu)^2 z_{eq}^{1/2} t_0 v_{\max}^{-6}. \quad (14)$$

Hence for values of  $R < R_{c_2}$ , there is no suppression from velocity effects and for  $R > R_{c_2}$  the suppression is given above by (13). Setting  $R_{c_2} = R_{c_1}$  we obtain a critical  $v_{\max}^c$  given by

$$v_{\max}^c = \left(\frac{1}{45}\right)^{1/3} \alpha^{1/2} \beta^{-1/6} \gamma^{-1/6} (G\mu)^{1/6} z_{eq}^{1/3}. \quad (15)$$

Thus, for  $v_{\max} < v_{\max}^c$  we have  $R_{c_2} > R_{c_1}$  and hence the mass function of predicted globular clusters from string loops will not change near the peak position compared to what was obtained in [1] neglecting the presence of string loop velocities. On the other hand, if  $v_{\max} > v_{\max}^c$  then the mass function will be suppressed near the peak position. The relation of  $v_{\max}^c$  and  $G\mu$  is illustrated in Fig. 2.

Taking into account the suppression factor when performing the calculations outlined in section II, we obtain the following histogram of predicted number of globular clusters:

$$\text{For } R_{c_2} > R_{c_1} : \quad (16)$$

$$\begin{aligned} \delta N &= N F f \alpha^{5/2} \beta^{-5/2} \gamma^{-3/2} \\ &\quad \times (G\mu)^{-3/2} z_{eq}^{3/2} t_0^{-3} V \quad \text{at } R_{c_1} \\ \delta N &= \left(\frac{1}{45}\right)^{-3} N F f \alpha^{-2} \beta^{-1} \\ &\quad \times (G\mu)^{-3} z_{eq}^{-3/2} t_0^{-3} v_{\max}^9 V \quad \text{at } R_{c_2} \end{aligned}$$

$$\text{For } R_{c_2} < R_{c_1} : \quad (17)$$

$$\begin{aligned} \delta N &= \frac{1}{45} N F f \alpha^4 \beta^{-3} \gamma^{-2} \\ &\quad \times (G\mu)^{-1} z_{eq}^{5/2} t_0^{-3} v_{\max}^{-3} V \quad \text{at } R_{c_1} \\ \delta N &= \left(\frac{1}{45}\right)^2 N F f \alpha^{11/2} \beta^{-7/2} \gamma^{-5/2} \\ &\quad \times (G\mu)^{-1/2} z_{eq}^{7/2} t_0^{-3} v_{\max}^{-6} V \quad \text{at } R_{c_2} \end{aligned}$$

Notice that for  $R_{c_1} < R_{c_2}$ , the mass scales as  $M^{-3}$  for  $R > R_{c_2}$ . In the radius  $R$  interval between  $R_{c_1}$  and  $R_{c_2}$  the mass function scales as  $M^{-5/2}$  as it does in the absence of velocity effects, and for masses smaller than  $M_c$ , a linear decay is predicted by the same reasoning as in section II. For  $R_{c_1} > R_{c_2}$ , the mass function scales as  $M^{-3}$  for  $R > R_{c_1}$ , as  $M^{-1/2}$  for  $R_{c_2} < R < R_{c_1}$  and decays linear for  $R < R_{c_2}$ .

In Fig. 3, we show that varying  $G\mu$  shifts the peak position and amplitude of the mass for a fixed  $v_{\max}^c$ . For  $R_{c_2} > R_{c_1}$ , initial velocity effects are negligible. However, for  $R_{c_2} < R_{c_1}$  but very close to the value of  $R_{c_1}$  there is a slight suppression in the region  $R_{c_2} < R < R_{c_1}$  from velocity effects.

In Fig. 4, we consider  $G\mu = 5.43 \times 10^{-10}$  which minimizes  $\chi^2$  in Fig. 1, we find from varying  $v_{\max}$  that for  $v_{\max} < 3.00 \times 10^{-2}$ , velocity has little effect on mass distribution of globular clusters in the Milky Way galaxy. However, for  $v_{\max} \gg 3.00 \times 10^{-2}$  we will not obtain a mass distribution.

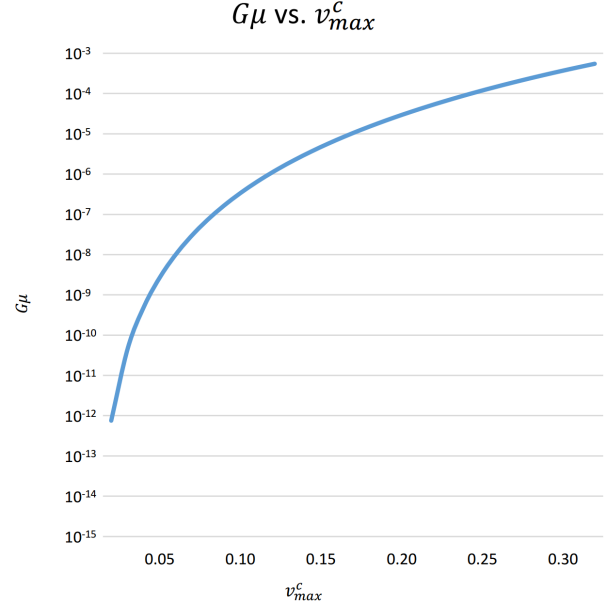


FIG. 2. Analysis 1 - Relation between  $G\mu$  and  $v_{\max}^c$  (15). The horizontal axis is velocity on a linear scale and the vertical axis gives the  $G\mu$  on a logarithmic scale.

#### IV. EFFECT OF COSMIC STRING VELOCITY: SECOND ANALYSIS

In the previous section we estimated the range of velocities for which spherical accretion onto a loop is a good approximation. On the other hand, accretion onto a moving loop can also be studied by means of the Zel'dovich approximation. This analysis is technically a bit more complicated than in the case of spherical accretion, but the study has been carried out in [14]. The result is that half of the turnaround mass from a string with some initial velocity is within a region which can be approximated by a paraboloid of radius  $r = b^{1/3} d_i$  and height  $h = 4b^{1/3} d_i$ , where (for loops born before the time of equal matter and radiation)

$$b(t) = \frac{1}{15} \frac{Gm}{v_{eq}^3 t_{eq}} a(t), \quad (18)$$

where  $m$  is the mass of the loop. Note that since the accretion effectively starts at  $t_{eq}$ , it is the loop velocity  $v_{eq}$  at that time which enters the formula.

The value of the mass enclosed in this region is:

$$M_{\text{ta}1/2}^{\text{ns}}(t) = \frac{3}{5} m a(t) \quad \text{for } b(t) \ll 1. \quad (19)$$

Assuming that the accreted mass has uniform density, we find that density  $\rho$  is given by

$$\rho = \frac{M}{V} = \frac{3}{10} \pi^{-1} b^{-1} d_i^{-3} m a(t) \quad \text{for } b(t) \ll 1. \quad (20)$$

Approximating the other half of the accreted mass to

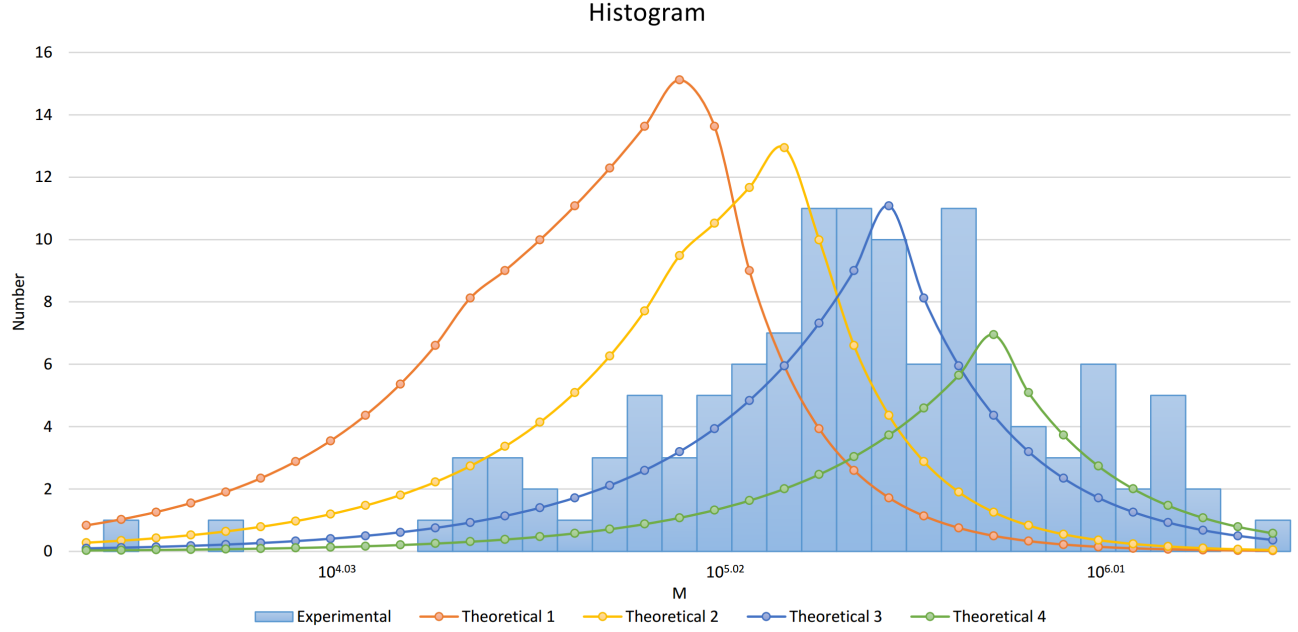


FIG. 3. Analysis 1 - Dependence of the mass function on  $G\mu$  at  $v_{\max} = 3.00 \times 10^{-2}$ . The horizontal axis is mass on a logarithmic scale, the vertical axis gives the number density on a linear scale. The histogram shows data taken from [12]. The curves shown are for  $G\mu = 2.92 \times 10^{-10}$ ,  $G\mu = 3.98 \times 10^{-10}$ ,  $G\mu = 5.43 \times 10^{-10}$ , and  $G\mu = 7.41 \times 10^{-10}$  (in increasing order of mass at the peak position). Notice that for the red and yellow solid curves,  $R_{c_2} < R_{c_1}$ , for the blue solid curve  $R_{c_2} = R_{c_1}$  and for the green solid curve  $R_{c_2} > R_{c_1}$ . The blue solid curve minimizes  $\chi^2$  for this particular  $v_{\max}$ .

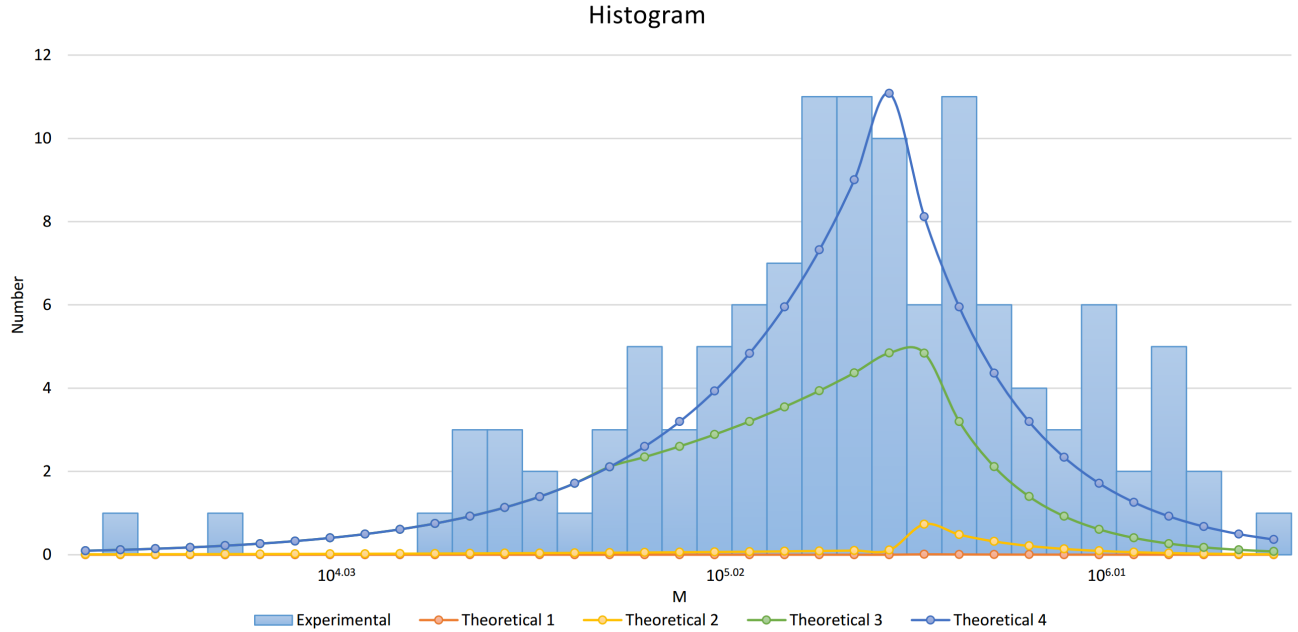


FIG. 4. Analysis 1 - Dependence of the mass function at  $G\mu = 5.43 \times 10^{-10}$  on  $v_{\max}$ . The curves are shown for  $v_{\max} = 1.00 \times 10^{-1}$  (Theoretical 1),  $v_{\max} = 6.46 \times 10^{-2}$  (Theoretical 2),  $v_{\max} = 3.44 \times 10^{-2}$  (Theoretical 3) and  $v_{\max} = 3.00 \times 10^{-2}$  (Theoretical 4). Notice that for all  $v_{\max} < 3.00 \times 10^{-2}$  we would obtain the blue curve. The axes and data are the same as in the previous figure.

be spherical, the total mass is given by

$$M_t^{\text{ns}} = \frac{4}{5}ma(t). \quad (21)$$

Comparing this to the mass from spherical accretion:

$$M_t^{\text{s}} = \frac{2}{5}ma(t) \quad (22)$$

we see non-spherical accretion results in a mass that is larger by a factor of two.

Accretion is roughly spherical when the loop accretion sphericity parameter  $b(t)$  as defined in (18) is larger than one. In this analysis, we will consider a slightly lower bound by setting  $b(t) > 10^{-1}$ . Using this condition, we obtain an equation for  $v_{eq}$ :

$$v_{eq} < \left(\frac{2}{3}\right)^{1/3} \beta^{1/3} (G\mu)^{1/3} z_{eq}^{5/6} t_0^{-1/3} R^{1/3}. \quad (23)$$

Red-shifting the velocity to the time of loop formation in the radiation dominated era we obtain:

$$v_i < \left(\frac{2}{3}\right)^{1/3} \alpha^{1/2} \beta^{-1/6} (G\mu)^{1/3} z_{eq}^{1/12} t_0^{1/6} R^{-1/6}. \quad (24)$$

Notice that the upper bound on the initial velocity in this analysis differs by only factor of  $10^{1/3}$  from the upper bound found in the first analysis. From here, performing the same steps as in Analysis 1 would obtain results that are larger by a factor  $10^{1/3}$ . This can be seen clearly by determining the new  $v_{\text{max}}^c$ :

$$v_{\text{max}}^c = \left(\frac{2}{9}\right)^{1/3} \alpha^{1/2} \beta^{-1/6} \gamma^{-1/6} (G\mu)^{1/6} z_{eq}^{1/3}. \quad (25)$$

The numerical results are presented in Fig. (5).

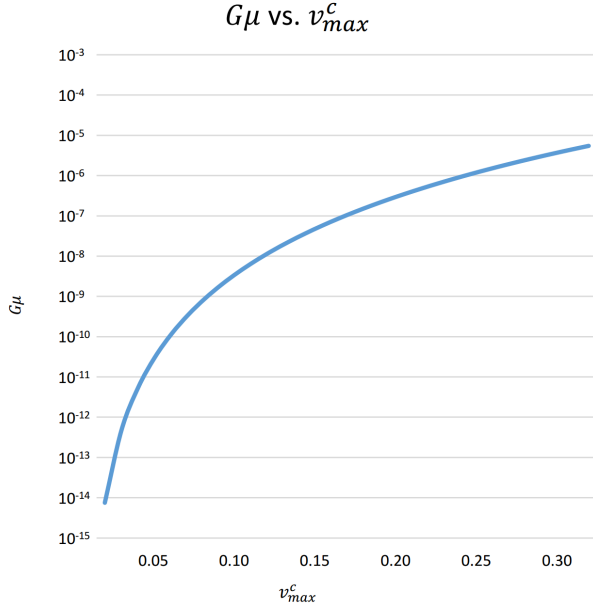


FIG. 5. Analysis 2 - Relation between  $G\mu$  and  $v_{\text{max}}^c$  (25). The horizontal axis is velocity on a linear scale and the vertical axis gives the  $G\mu$  on a logarithmic scale.

## V. THE ROCKET EFFECT

The *rocket effect* is an effect which causes the string loop to accelerate as a consequence of anisotropic gravitational radiation. Let us assume that the string loop has a center of mass velocity  $v_{eq}$  at  $t_{eq}$ . Hubble expansion will lead to a decrease in the velocity whereas the anisotropic gravitational radiation will tend to cause an increase. Which effect wins depends on time.

The equation of motion for the loop velocity taking into account Hubble expansion and anisotropic gravitational radiation is (see e.g. Appendix A of [15] - we have generalized the result to times before  $t_{eq}$ )

$$v(t) = v_i \frac{z(t)}{z(t_i)} + \left( \frac{3\Gamma_P (G\mu)^2 M_i}{5m} \right) \left( \frac{t}{t_i} - \frac{t_i^{1/2}}{t^{1/2}} \right), \quad (26)$$

where  $\Gamma_P$  is the coefficient of anisotropic gravitational radiation,  $M_i$  is the mass at the loop formation time  $t_i$  and  $m$  is the mass at a later time. We will take  $\Gamma_P \simeq 10$ . This value was estimated by numerical simulations using the Kibble-Turok loop solutions [16].

We now ask the question whether the rocket effect can have an important effect on gravitational accretion by the loops which dominate the globular cluster mass function. Thus, we are interested in loops with initial radius  $R = \gamma G\mu t_{eq}$ . If the second term on the right-hand side of (26) is smaller than the first term at the time  $t = t_{eq}$ , then we argue that the rocket effect will be negligible since even if at very late times the string loop starts to accelerate away from the initial position of the loop, the nonlinear object seeded by the loop is already in place one Hubble time after  $t_{eq}$ . Making use of (26) we then obtain the condition

$$v_i (G\mu)^{-1/2} > \frac{3}{5} \Gamma_P \alpha^{3/2} \beta^{-3/2} \gamma^{-3/2}, \quad (27)$$

which, if satisfied, guarantees that the rocket effect is negligible. But for the values of  $G\mu$  smaller than the current observational bound this condition is obviously satisfied. Hence, we conclude that the rocket effect does not effect our analysis.

## VI. CONCLUSIONS AND DISCUSSION

In this paper we have considered the effects of string loop velocities on the model for the explanation of the origin of globular clusters proposed in [1] in which it is proposed that globular clusters are seeded by cosmic string loops.

We see that velocities play a small, but noticeable role in the accretion of matter during the matter-dominated era. Cosmic string loops, born in the radiation era at  $t_i$  with initial velocity  $v_i$  will travel a certain given by (9). Demanding that this distance be smaller than the total size of the structure which accretes around a static loop, denoted in this paper by  $h(R, t_0)$ , leads to an upper

bound on the string velocities. In our Analysis 1 we assumed that loops with larger velocities do not give rise to globular clusters. We computed the corrections to the mass function obtained using this hypothesis, as a function of the maximal velocity of string loops. Our theory predicts a cutoff velocity. For velocities smaller than  $v_{\text{max}}^c$ , velocity effects are negligible.

An improved analysis (Analysis 2) can be obtained by taking into account the non-spherical accretion of matter about a moving string loop, and demanding that the resulting nonlinear object is sufficiently spherical. The results of this analysis were very similar to those obtained using Analysis 1.

Finally, we have shown that the *rocket effect* does not

affect our analysis.

Aside from globular clusters, it is noteworthy to mention that analyses discussed in this report may also be applied to ultra-compact-mini-halos, as they too can be manifestations of cosmic string loop accretion [17]. Work on this topic is in progress.

## ACKNOWLEDGMENTS

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- [1] A. Barton, R. H. Brandenberger and L. Lin, “Cosmic Strings and the Origin of Globular Clusters,” JCAP **1506**, no. 06, 022 (2015) [arXiv:1502.07301 [astro-ph.CO]].
  - [2] T. W. B. Kibble, “Phase Transitions In The Early Universe,” Acta Phys. Polon. B **13**, 723 (1982);  
T. W. B. Kibble, “Some Implications Of A Cosmological Phase Transition,” Phys. Rept. **67**, 183 (1980).
  - [3] N. Turok and R. H. Brandenberger, “Cosmic Strings And The Formation Of Galaxies And Clusters Of Galaxies,” Phys. Rev. D **33**, 2175 (1986);  
H. Sato, “Galaxy Formation by Cosmic Strings,” Prog. Theor. Phys. **75**, 1342 (1986);  
A. Stebbins, “Cosmic Strings and Cold Matter”, Ap. J. (Lett.) **303**, L21 (1986).
  - [4] C. Dvorkin, M. Wyman and W. Hu, “Cosmic String constraints from WMAP and the South Pole Telescope,” Phys. Rev. D **84**, 123519 (2011) [arXiv:1109.4947 [astro-ph.CO]];  
P. A. R. Ade *et al.* [Planck Collaboration], “Planck 2013 results. XXV. Searches for cosmic strings and other topological defects,” Astron. Astrophys. **571**, A25 (2014) [arXiv:1303.5085 [astro-ph.CO]].
  - [5] L. Pogosian, S. H. H. Tye, I. Wasserman and M. Wyman, “Observational constraints on cosmic string production during brane inflation,” Phys. Rev. D **68**, 023506 (2003) [Erratum-ibid. D **73**, 089904 (2006)] [arXiv:hep-th/0304188];  
M. Wyman, L. Pogosian and I. Wasserman, “Bounds on cosmic strings from WMAP and SDSS,” Phys. Rev. D **72**, 023513 (2005) [Erratum-ibid. D **73**, 089905 (2006)] [arXiv:astro-ph/0503364];  
A. A. Fraisse, “Limits on Defects Formation and Hybrid Inflationary Models with Three-Year WMAP Observations,” JCAP **0703**, 008 (2007) [arXiv:astro-ph/0603589];  
U. Seljak, A. Slosar and P. McDonald, “Cosmological parameters from combining the Lyman-alpha forest with CMB, galaxy clustering and SN constraints,” JCAP **0610**, 014 (2006) [arXiv:astro-ph/0604335];  
R. A. Battye, B. Garbrecht and A. Moss, “Constraints on supersymmetric models of hybrid inflation,” JCAP **0609**, 007 (2006) [arXiv:astro-ph/0607339];  
R. A. Battye, B. Garbrecht, A. Moss and H. Stoica, “Constraints on Brane Inflation and Cosmic Strings,” JCAP **0801**, 020 (2008) [arXiv:0710.1541 [astro-ph]];  
N. Bevis, M. Hindmarsh, M. Kunz and J. Urrestilla, “CMB power spectrum contribution from cosmic strings using field-evolution simulations of the Abelian Higgs model,” Phys. Rev. D **75**, 065015 (2007) [arXiv:astro-ph/0605018];  
N. Bevis, M. Hindmarsh, M. Kunz and J. Urrestilla, “Fitting CMB data with cosmic strings and inflation,” Phys. Rev. Lett. **100**, 021301 (2008) [astro-ph/0702223 [ASTRO-PH]];  
R. Battye and A. Moss, “Updated constraints on the cosmic string tension,” Phys. Rev. D **82**, 023521 (2010) [arXiv:1005.0479 [astro-ph.CO]].
  - [6] A. Albrecht and N. Turok, “Evolution Of Cosmic Strings,” Phys. Rev. Lett. **54**, 1868 (1985);  
D. P. Bennett and F. R. Bouchet, “Evidence For A Scaling Solution In Cosmic String Evolution,” Phys. Rev. Lett. **60**, 257 (1988);  
B. Allen and E. P. S. Shellard, “Cosmic String Evolution: A Numerical Simulation,” Phys. Rev. Lett. **64**, 119 (1990);  
C. Ringeval, M. Sakellariadou and F. Bouchet, “Cosmological evolution of cosmic string loops,” JCAP **0702**, 023 (2007) [arXiv:astro-ph/0511646];  
V. Vanchurin, K. D. Olum and A. Vilenkin, “Scaling of cosmic string loops,” Phys. Rev. D **74**, 063527 (2006) [arXiv:gr-qc/0511159];  
L. Lorenz, C. Ringeval and M. Sakellariadou, “Cosmic string loop distribution on all length scales and at any redshift,” JCAP **1010**, 003 (2010) [arXiv:1006.0931 [astro-ph.CO]];  
J. J. Blanco-Pillado, K. D. Olum and B. Shlaer, “Large parallel cosmic string simulations: New results on loop production,” Phys. Rev. D **83**, 083514 (2011) [arXiv:1101.5173 [astro-ph.CO]];  
J. J. Blanco-Pillado, K. D. Olum and B. Shlaer, “The number of cosmic string loops,” Phys. Rev. D **89**, no. 2, 023512 (2014) [arXiv:1309.6637 [astro-ph.CO]].
  - [7] A. Vilenkin, “Cosmic Strings,” Phys. Rev. D **24**, 2082 (1981).
  - [8] T. W. B. Kibble, “Evolution of a system of cosmic strings,” Nucl. Phys. B **252**, 227 (1985) [Nucl. Phys. B **261**, 750 (1985)].
  - [9] T. Vachaspati and A. Vilenkin, “Gravitational Radiation from Cosmic Strings,” Phys. Rev. D **31**, 3052

- (1985).
- [10] A. Vilenkin and E.P.S. Shellard, *Cosmic Strings and other Topological Defects* (Cambridge Univ. Press, Cambridge, 1994);  
M. B. Hindmarsh and T. W. B. Kibble, “Cosmic strings,” Rept. Prog. Phys. **58**, 477 (1995) [arXiv:hep-ph/9411342];  
R. H. Brandenberger, “Topological defects and structure formation,” Int. J. Mod. Phys. A **9**, 2117 (1994) [arXiv:astro-ph/9310041].
  - [11] Y. .B. Zeldovich, “Gravitational instability: An Approximate theory for large density perturbations,” Astron. Astrophys. **5**, 84 (1970).
  - [12] O. Y. Gnedin and J. P. Ostriker, “Destruction of the galactic globular cluster system,” Astrophys. J. **474**, 223 (1997) [astro-ph/9603042].
  - [13] M. Pagano and R. Brandenberger, “The 21cm Signature of a Cosmic String Loop,” JCAP **1205**, 014 (2012) [arXiv:1201.5695 [astro-ph.CO]].
  - [14] E. Bertschinger, “Cosmological Accretion Wakes”, Astrophys. J. **316**, 489 (1987).
  - [15] B. Shlaer, A. Vilenkin and A. Loeb, “Early structure formation from cosmic string loops,” JCAP **1205**, 026 (2012) [arXiv:1202.1346 [astro-ph.CO]].
  - [16] T. W. B. Kibble and N. Turok, “Selfintersection of Cosmic Strings,” Phys. Lett. B **116**, 141 (1982).
  - [17] M. Anthonisen, R. Brandenberger and P. Scott, “Constraints on cosmic strings from ultracompact minihalos,” Phys. Rev. D **92**, no. 2, 023521 (2015) [arXiv:1504.01410 [astro-ph.CO]].